

US-FT-11/94
July, 1994

Superconformal current algebras and topological field theories

J. M. ISIDRO[★]

*Department of Physics, Queen Mary and Westfield College,
Mile End Road, London E1 4NS, United Kingdom.*

A.V. RAMALLO

*Departamento de Física de Partículas,
Universidad de Santiago,
E-15706 Santiago de Compostela, Spain.*

ABSTRACT

Topological conformal field theories based on superconformal current algebras are constructed. The models thus obtained are the supersymmetric version of the G/G coset theories. Their topological conformal algebra is generated by operators of dimensions 1, 2 and 3 and can be regarded as an extension of the twisted $N = 2$ superconformal algebra. These models possess an extended supersymmetry whose generators are exact in the topological BRST cohomology.

★ On leave from : Departamento de Física de Partículas, Universidad de Santiago, E-15706 Santiago de Compostela, Spain.

Topological field theories [1,2] are quantum field theories that do not possess any local degree of freedom. They are endowed with a topological symmetry that allows to eliminate all local excitations, in such a way that only the collective modes of the basic fields remain in its spectrum after the topological symmetry is fixed. Some models can be converted into topological theories by adding a ghost sector and a BRST symmetry that can be used to define a cohomology. In so doing one expects to obtain some non-trivial information about the non-perturbative sector of the initial theory. We have recently [3,4] applied this procedure to generate two-dimensional topological conformal field theories based on non-abelian current algebras. In a model with a current algebra symmetry, the local degrees of freedom are created by acting with the currents on the vacuum. The BRST symmetry studied in [3,4] is such that the total Kac-Moody current is BRST-exact (*i.e.*, cohomologically trivial). Moreover, we have shown that the topological conformal algebra contains operators of dimensions 1, 2 and 3 and can be considered as an extension of the twisted [5] $N = 2$ superconformal algebra [6].

In this paper we propose to construct topological theories based on superconformal current algebras [7,8]. Let us consider a finite-dimensional, semisimple Lie algebra g , generated by the hermitian matrices T^a ($a = 1, \dots, \dim g$). The T^a are chosen such that $\text{Tr}(T^a T^b) = \delta_{ab}$ and to satisfy the commutation relations

$$[T^a, T^b] = i f^{abc} T^c. \quad (1)$$

A superconformal current algebra is generated by a set of holomorphic bosonic currents $\mathcal{J}_a(z)$ and their fermionic partners $\Phi_a(z)$, satisfying the following operator product expansions (OPE's):

$$\begin{aligned} \mathcal{J}_a(z_1) \mathcal{J}_b(z_2) &= \frac{x \delta_{ab}}{(z_1 - z_2)^2} + i f^{abc} \frac{\mathcal{J}_c(z_2)}{z_1 - z_2} \\ \mathcal{J}_a(z_1) \Phi_b(z_2) &= i f^{abc} \frac{\Phi_c(z_2)}{z_1 - z_2} \\ \Phi_a(z_1) \Phi_b(z_2) &= \frac{x \delta_{ab}}{z_1 - z_2}, \end{aligned} \quad (2)$$

where x is the level of the current algebra. \mathcal{J}_a and Φ_a are primary fields with respect to the energy-momentum tensor T with conformal dimensions 1 and $\frac{1}{2}$ respectively. This model is supersymmetric, which implies that a dimension- $\frac{3}{2}$ fermionic operator T_F exists with the OPE

$$T_F(z_1)T_F(z_2) = \frac{c}{6(z_1 - z_2)^3} + \frac{1}{2} \frac{T(z_2)}{z_1 - z_2}, \quad (3)$$

where c is the central charge of the Virasoro algebra. The currents \mathcal{J}_a and Φ_a form a doublet under the supersymmetry algebra generated by T_F , so we must have

$$\begin{aligned} T_F(z_1)\Phi_a(z_2) &= \frac{1}{2} \frac{\mathcal{J}_a(z_2)}{z_1 - z_2} \\ T_F(z_1)\mathcal{J}_a(z_2) &= \frac{1}{2} \frac{\Phi_a(z_2)}{(z_1 - z_2)^2} + \frac{1}{2} \frac{\partial\Phi_a(z_2)}{z_1 - z_2}. \end{aligned} \quad (4)$$

It is possible to realize this superconformal algebra in terms of a system of uncoupled bosonic currents j_a and $\dim g$ free Majorana fermions ψ_a transforming in the adjoint representation of g , the corresponding OPE's being

$$\begin{aligned} j_a(z_1)j_b(z_2) &= \frac{k\delta_{ab}}{(z_1 - z_2)^2} + if^{abc} \frac{j_c(z_2)}{z_1 - z_2} \\ \psi_a(z_1)\psi_b(z_2) &= - \frac{\delta_{ab}}{z_1 - z_2}. \end{aligned} \quad (5)$$

The energy-momentum tensor T is obtained by combining the Sugawara form of the bosonic sector with the canonical energy-momentum tensor of the Majorana fermions^{*},

$$T^{(j,\psi)} = \frac{1}{2(k+h)} j_a j_a + \frac{1}{2} \psi_a \partial\psi_a, \quad (6)$$

where h is the dual Coxeter number of g . For simply-laced algebras $h = \dim g / \text{rank } g - 1$ (*i.e.*, for instance, $h = N$ for $g = sl(N)$). $T^{(j,\psi)}$ closes a Vi-

* In the following, although we will not denote it explicitly, all products of fields will be understood as normal-ordered.

rasoro algebra with central charge

$$c^{(j,\psi)} = \frac{k \dim g}{k+h} + \frac{1}{2} \dim g . \quad (7)$$

Now it's easy to check that the supersymmetry generator T_F satisfying (3) is

$$T_F^{(j,\psi)} = \frac{i}{2(k+h)^{\frac{1}{2}}} j_a \psi_a - \frac{1}{12(k+h)^{\frac{1}{2}}} f^{abc} \psi_a \psi_b \psi_c , \quad (8)$$

and that the currents

$$\begin{aligned} \mathcal{J}_a^{(j,\psi)} &= j_a + \frac{i}{2} f^{abc} \psi_b \psi_c \\ \Phi_a^{(j,\psi)} &= i(k+h)^{\frac{1}{2}} \psi_a \end{aligned} \quad (9)$$

close the superconformal current algebra (2) with level

$$x^{(j,\psi)} = k+h . \quad (10)$$

Moreover it can be readily verified that \mathcal{J}_a and Φ_a form a doublet with respect to $T_F^{(j,\psi)}$, *i.e.* \mathcal{J}_a and Φ_a as given by eq. (9) satisfy (4).

The superconformal current algebra (2) can also be realized in a supersymmetric ghost system. Let us introduce a pair of anticommuting ghosts γ_a and ρ_a with dimensions 0 and 1 respectively. The supersymmetry requirement leads us to introduce commuting ghosts η_a and λ_a , both of them with dimension $\frac{1}{2}$. To the fields γ_a and η_a (ρ_a and λ_a) we shall assign ghost number +1 (−1 respectively). Let us choose our conventions in such a way that the ghost fields obey the OPE's

$$\begin{aligned} \rho_a(z_1) \gamma_b(z_2) &= - \frac{\delta_{ab}}{z_1 - z_2} \\ \eta_a(z_1) \lambda_b(z_2) &= - \frac{\delta_{ab}}{z_1 - z_2} . \end{aligned} \quad (11)$$

Since the canonical energy-momentum tensor of this system is

$$T^{(gh)} = \rho_a \partial \gamma_a + \frac{1}{2} \partial \eta_a \lambda_a - \frac{1}{2} \eta_a \partial \lambda_a , \quad (12)$$

the (γ, ρ) and (η, λ) systems contribute respectively with $-2 \dim g$ and $-\dim g$ to

the Virasoro central charge. Therefore their conformal anomaly is

$$c^{(gh)} = -3 \dim g . \quad (13)$$

The supersymmetry in this ghost system is generated by

$$T_F^{(gh)} = \frac{1}{2} \eta_a \rho_a - \frac{1}{2} \partial \gamma_a \lambda_a , \quad (14)$$

and the explicit form of the superconformal currents is

$$\begin{aligned} \mathcal{J}_a^{(gh)} &= i f^{abc} \gamma_b \rho_c + i f^{abc} \lambda_b \eta_c \\ \Phi_a^{(gh)} &= i f^{abc} \gamma_b \lambda_c . \end{aligned} \quad (15)$$

In this case the algebra (2) is closed with vanishing level, *i.e.*, one has

$$x^{(gh)} = 0 . \quad (16)$$

Let us now combine the matter and ghost realizations we have described above. Suppose we consider M copies of the matter system, together with one copy of the supersymmetric ghost model, and let us denote by j_a^l and ψ_a^l ($l = 1, \dots, M$) the basic fields of the matter sector. The currents of the combined matter + ghost system are given by

$$\begin{aligned} \mathcal{J}_a &= \sum_{l=1}^M \left(j_a^l + \frac{i}{2} f^{abc} \psi_b^l \psi_c^l \right) + i f^{abc} \gamma_b \rho_c + i f^{abc} \lambda_b \eta_c \\ \Phi_a &= i \sum_{l=1}^M (k_l + h)^{\frac{1}{2}} \psi_a^l + i f^{abc} \gamma_b \lambda_c , \end{aligned} \quad (17)$$

where k_l is the level of the current j_a^l . The complete energy-momentum tensor now

takes the form

$$T = \sum_{l=1}^M \frac{1}{2(k_l + h)} j_a^l j_a^l + \frac{1}{2} \sum_{l=1}^M \psi_a^l \partial \psi_a^l + \rho_a \partial \gamma_a + \frac{1}{2} \partial \eta_a \lambda_a - \frac{1}{2} \eta_a \partial \lambda_a , \quad (18)$$

and the central charge c and level x are given by

$$\begin{aligned} c &= \sum_{l=1}^M \frac{k_l \dim g}{k_l + h} + \left(\frac{M}{2} - 3 \right) \dim g \\ x &= \sum_{l=1}^M k_l + M h . \end{aligned} \quad (19)$$

We would like this combined matter + ghost system to be a topological theory. As it was shown in [3], a way of finding the appropriate content of the matter sector is to impose the zero level condition $x = 0$. If we had only one matter species (*i.e.*, for $M = 1$), $x = 0$ would mean $k_1 + h = 0$ (see eq. (19)). However the value $k_1 = -h$ is not acceptable since in this case the expression of the energy-momentum tensor T in (18) becomes singular. Let us therefore consider the case $M = 2$. With two currents in the matter sector, the condition $x = 0$ implies

$$k_1 + k_2 = -2h . \quad (20)$$

An indication that we are now pointing in the right direction is the fact that the central charge c vanishes when eq. (20) holds (see (19)), which confirms that we have found a topological point of the matter + ghost system. In fact, when the two levels k_1 and k_2 are related as in (20), the $N = 1$ supersymmetry is extended to a larger algebra that we are now going to describe. The generators of this extended algebra are naturally expressed in terms of the following complex combinations of the two real Majorana fields ψ_a^1 and ψ_a^2 :

$$\Psi_a = \frac{1}{\sqrt{2}}(\psi_a^1 + i\psi_a^2) \quad \bar{\Psi}_a = \frac{1}{\sqrt{2}}(\psi_a^1 - i\psi_a^2) , \quad (21)$$

so from (5) one gets

$$\begin{aligned}\Psi_a(z_1)\Psi_b(z_2) &= \bar{\Psi}_a(z_1)\bar{\Psi}_b(z_2) = 0 \\ \Psi_a(z_1)\bar{\Psi}_b(z_2) &= -\frac{\delta_{ab}}{z_1 - z_2} .\end{aligned}\tag{22}$$

Similarly, let us also define the combinations of the bosonic currents

$$J_a = j_a^1 + j_a^2 \quad \bar{J}_a = j_a^1 - j_a^2 ,\tag{23}$$

whose OPE's are

$$\begin{aligned}J_a(z_1)J_b(z_2) &= \bar{J}_a(z_1)\bar{J}_b(z_2) = -\frac{2h}{(z_1 - z_2)^2} \delta_{ab} + if^{abc} \frac{J_c(z_2)}{z_1 - z_2} \\ J_a(z_1)\bar{J}_b(z_2) &= \frac{2(\kappa + h)}{(z_1 - z_2)^2} \delta_{ab} + if^{abc} \frac{\bar{J}_c(z_2)}{z_1 - z_2} .\end{aligned}\tag{24}$$

We have written κ instead of k_1 (and therefore $k_2 = -2h - \kappa$). As $(k_2 + h)^{\frac{1}{2}} = i(\kappa + h)^{\frac{1}{2}}$, in terms of these new variables the superconformal currents take the form

$$\begin{aligned}\mathcal{J}_a &= J_a + if^{abc} \bar{\Psi}_b \Psi_c + if^{abc} \lambda_b \eta_c + if^{abc} \gamma_b \rho_c \\ \Phi_a &= i\sqrt{2(\kappa + h)} \Psi_a + if^{abc} \gamma_b \lambda_c .\end{aligned}\tag{25}$$

Consider now the following dimension- $\frac{3}{2}$ operators:

$$\begin{aligned}T_F^+ &= \frac{1}{\sqrt{2(\kappa + h)}} (iJ_a \bar{\Psi}_a - \frac{1}{2} f^{abc} \bar{\Psi}_a \bar{\Psi}_b \Psi_c) + \eta_a \rho_a \\ T_F^- &= \frac{i}{\sqrt{2(\kappa + h)}} \bar{J}_a \Psi_a - \partial \gamma_a \lambda_a ,\end{aligned}\tag{26}$$

which, as can be easily checked, are primary with respect to T and satisfy the algebra

$$\begin{aligned}T_F^+(z_1)T_F^+(z_2) &= 0 \\ T_F^+(z_1)T_F^-(z_2) &= \frac{R_F(z_2)}{(z_1 - z_2)^2} + \frac{T(z_2) + \frac{1}{2}\partial R_F(z_2)}{z_1 - z_2} \\ T_F^-(z_1)T_F^-(z_2) &= \frac{W_F(z_2)}{z_1 - z_2} .\end{aligned}\tag{27}$$

R_F and W_F are bosonic operators of dimensions 1 and 2 respectively, whose explicit

expressions are

$$\begin{aligned} R_F &= \Psi_a \bar{\Psi}_a + \eta_a \lambda_a \\ W_F &= \frac{1}{\kappa + h} (h \partial \Psi_a \Psi_a - \frac{i}{2} f^{abc} \Psi_a \Psi_b J_c) . \end{aligned} \quad (28)$$

The algebra of T , T_F^\pm , R_F and W_F closes only after the introduction of a new dimension- $\frac{3}{2}$ operator V_F :

$$V_F = \frac{1}{3\sqrt{2(\kappa + h)}} f^{abc} \Psi_a \Psi_b \Psi_c . \quad (29)$$

It can be readily verified that R_F , W_F and V_F are primary fields. In fact V_F shows up when acting on W_F with T_F^- . One has :

$$\begin{aligned} T_F^+(z_1) W_F(z_2) &= 0 \\ T_F^-(z_1) W_F(z_2) &= \frac{3}{(z_1 - z_2)^2} V_F(z_2) + \frac{1}{z_1 - z_2} \partial V_F(z_2) \\ T_F^+(z_1) V_F(z_2) &= \frac{W_F(z_2)}{z_1 - z_2} \\ T_F^-(z_1) V_F(z_2) &= 0 . \end{aligned} \quad (30)$$

The OPE's between W_F and V_F vanish, namely

$$W_F(z_1) W_F(z_2) = W_F(z_1) V_F(z_2) = V_F(z_1) V_F(z_2) = 0 , \quad (31)$$

and the product of R_F with itself is non-singular; in fact, R_F introduces a conserved $U(1)$ charge. By inspection one verifies that all the generators of the extended supersymmetry algebra have a well-defined R_F -charge, which is equal to $+1$, -1 , -2 and -3 for T_F^+ , T_F^- , W_F and V_F respectively.

We shall call the algebra displayed in eqs. (27), (30) and (31) the *supersymmetric Kazama algebra*; it was first introduced in [9]. In our case this algebra is realized with a vanishing central charge. Notice that, for g abelian, W_F and V_F vanish identically and the supersymmetric Kazama algebra is nothing but the usual $N = 2$ superconformal algebra. Moreover the matter and ghost parts of our

model separately realize the supersymmetry algebra with central charges $3 \dim g$ and $-3 \dim g$ respectively (actually the ghost system has an $N = 2$ supersymmetry). On the other hand, the form of the generators in the matter sector displayed in eqs. (26), (28) and (29) can be obtained by twisting the realization of the extended topological algebras found in refs.[3,4] (see also ref. [10] where this algebra is termed $N = 1\frac{1}{2}$ superconformal algebra). The operator T_F that generates the $N = 1$ subalgebra can be obtained as a particular combination of the three dimension- $\frac{3}{2}$ generators:

$$T_F = \frac{1}{2}(T_F^+ + T_F^- - \frac{1}{2}V_F) . \quad (32)$$

It can also be checked that T_F^\pm and R_F act on the superconformal currents \mathcal{J}_a and Φ_a as follows:

$$\begin{aligned} T_F^+(z_1)\mathcal{J}_a(z_2) &= 0 \\ T_F^-(z_1)\mathcal{J}_a(z_2) &= \frac{\Phi_a(z_2)}{(z_1 - z_2)^2} + \frac{\partial\Phi_a(z_2)}{z_1 - z_2} \\ R_F(z_1)\mathcal{J}_a(z_2) &= 0 \\ T_F^+(z_1)\Phi_a(z_2) &= \frac{\mathcal{J}_a(z_2)}{z_1 - z_2} \\ T_F^-(z_1)\Phi_a(z_2) &= 0 \\ R_F(z_1)\Phi_a(z_2) &= \frac{-1}{z_1 - z_2}\Phi_a(z_2) . \end{aligned} \quad (33)$$

The OPE's of W_F and V_F with the currents vanish. This fact, together with (33), shows the compatibility of the supersymmetric and current algebras.

Let us now show that this system possesses a BRST symmetry such that the currents \mathcal{J}_a and Φ_a become BRST-exact. We have at our disposal two antighost fields ρ_a and λ_a with conformal dimensions 1 and $\frac{1}{2}$ respectively, so it's natural to require to our BRST transformation δ that

$$\delta\rho_a = \mathcal{J}_a \quad \delta\lambda_a = \Phi_a . \quad (34)$$

Using eqs. (11), (22) y (24), it is easy to check that the transformations in (34)

are obtained by acting with the zero mode of the operator

$$Q = -\gamma_a(J_a + if^{abc} \bar{\Psi}_b \Psi_c + if^{abc} \lambda_b \eta_c + \frac{i}{2} f^{abc} \gamma_b \rho_c) - i\sqrt{2(\kappa + h)} \eta_a \Psi_a , \quad (35)$$

the δ -variations of the other fields being

$$\begin{aligned} \delta\gamma_a &= \frac{i}{2} f^{abc} \gamma_b \gamma_c \\ \delta\eta_a &= if^{abc} \gamma_b \eta_c \\ \delta J_a &= if^{abc} \gamma_b J_c + 2h\partial\gamma_a \\ \delta\bar{J}_a &= if^{abc} \gamma_b \bar{J}_c - 2(\kappa + h)\partial\gamma_a \\ \delta\Psi_a &= if^{abc} \gamma_b \Psi_c \\ \delta\bar{\Psi}_a &= if^{abc} \gamma_b \bar{\Psi}_c + i\sqrt{2(\kappa + h)} \eta_a . \end{aligned} \quad (36)$$

A standard calculation shows that $Q(z)$ is a nilpotent operator:

$$Q(z_1)Q(z_2) = 0 . \quad (37)$$

In order to show that Q is a generator of a topological symmetry we must prove that the energy-momentum tensor T can be written as a BRST variation. Let us call G the BRST partner of T , *i.e.*, the dimension-2 field such that $T = \delta G$. The expression of G is more conveniently written in terms of the operator

$$\Lambda_a = \rho_a - \frac{f^{abc}}{\sqrt{2(\kappa + h)}} \lambda_b \bar{\Psi}_c , \quad (38)$$

which is a dimension-1 operator with ghost number -1 . Using eqs. (34) and (35) it can be readily verified that its BRST variation is

$$\delta\Lambda_a = J_a + if^{abc} \gamma_b \Lambda_c . \quad (39)$$

In terms of Λ_a , we now define G as

$$G = \frac{1}{2(\kappa + h)} \bar{J}_a \Lambda_a + \frac{i}{2\sqrt{2(\kappa + h)}} (\partial\lambda_a \bar{\Psi}_a - \lambda_a \partial\bar{\Psi}_a) . \quad (40)$$

Indeed, one can check that the OPE of Q and G is

$$Q(z_1)G(z_2) = \frac{T(z_2)}{z_1 - z_2} + \frac{R(z_2)}{(z_1 - z_2)^2} + \frac{d}{(z_1 - z_2)^3} , \quad (41)$$

where

$$\begin{aligned} d &= \dim g \\ R &= \rho_a \gamma_a + \frac{1}{2} \eta_a \lambda_a + \frac{1}{2} \bar{\Psi}_a \Psi_a . \end{aligned} \quad (42)$$

Eq. (41) is the first in a number of OPE's characterising the topological symmetry of this model. Other OPE's involving T , Q , R and G are

$$\begin{aligned} T(z_1)Q(z_2) &= \frac{Q(z_2)}{(z_1 - z_2)^2} + \frac{\partial Q(z_2)}{z_1 - z_2} \\ T(z_1)R(z_2) &= -\frac{d}{(z_1 - z_2)^3} + \frac{R(z_2)}{(z_1 - z_2)^2} + \frac{\partial R(z_2)}{z_1 - z_2} \\ T(z_1)G(z_2) &= \frac{2G(z_2)}{(z_1 - z_2)^2} + \frac{\partial G(z_2)}{z_1 - z_2} \\ R(z_1)R(z_2) &= \frac{d}{(z_1 - z_2)^2} \\ R(z_1)Q(z_2) &= \frac{Q(z_2)}{z_1 - z_2} \\ R(z_1)G(z_2) &= -\frac{G(z_2)}{z_1 - z_2} . \end{aligned} \quad (43)$$

Notice that, according to eq.(43), Q and G are primary fields with dimensions 1 and 2 respectively, whereas R is an anomalous $U(1)$ current, d being the corresponding anomaly. We shall call d the *dimension of the topological algebra*; in our case d equals precisely the dimension of the Lie algebra g . Curiously, the same value of d is obtained in the non-supersymmetric topological current algebras [3].

An important feature of the topological algebra just described is the fact that G is *not* a nilpotent operator. Actually the OPE of G with itself gives rise to a

new dimension-3 operator W :

$$G(z_1)G(z_2) = \frac{W(z_2)}{z_1 - z_2} , \quad (44)$$

where W is a commuting field given by

$$W = \frac{1}{[2(\kappa + h)]^2} (i f^{abc} J_a \Lambda_b \Lambda_c - 2h \partial \Lambda_a \Lambda_a) . \quad (45)$$

This W operator is BRST-exact: its BRST ancestor is

$$V = \frac{i}{3[2(\kappa + h)]^2} f^{abc} \Lambda_a \Lambda_b \Lambda_c , \quad (46)$$

which is an anticommuting dimension-3 operator. Acting with Q on V one gets a simple pole with W as residue:

$$Q(z_1)V(z_2) = \frac{W(z_2)}{z_1 - z_2} . \quad (47)$$

After the introduction of these two new fields, the topological algebra generated by Q , R , G , T , V and W closes. Apart from those already displayed in eqs. (41), (43), (44) and (47), the non-vanishing OPE's are

$$\begin{aligned} R(z_1)W(z_2) &= -\frac{2}{z_1 - z_2} W(z_2) \\ T(z_1)W(z_2) &= \frac{3}{(z_1 - z_2)^2} W(z_2) + \frac{\partial W(z_2)}{z_1 - z_2} \\ G(z_1)W(z_2) &= \frac{3}{(z_1 - z_2)^2} V(z_2) + \frac{\partial V(z_2)}{z_1 - z_2} \\ R(z_1)V(z_2) &= -\frac{3}{z_1 - z_2} V(z_2) \\ T(z_1)V(z_2) &= \frac{3}{(z_1 - z_2)^2} V(z_2) + \frac{\partial V(z_2)}{z_1 - z_2} . \end{aligned} \quad (48)$$

Therefore the topological symmetry of our model is generated by three BRST doublets of operators $((R, Q)$, (G, T) and (V, W)) of dimensions 1, 2 and 3. It is

important to point out that the topological algebra we have obtained is *not* the twisted $N = 2$ algebra; in fact what we have is an extended topological algebra that can be obtained by twisting a supersymmetric Kazama algebra of the type described above. We have obtained another realization of this extended topological symmetry in refs. [3,4] in our study of the topological conformal field theories possessing a bosonic, non-abelian current algebra (see also [10]). The G/G coset theories [11,12,13,14] are particular cases of these models. It is interesting to observe that the same algebra appears when one requires the topological theory to have a superconformal current symmetry.

Let us now study the compatibility of the topological symmetry and the superconformal current algebra. By inspecting the form of the topological $U(1)$ current (*i.e.*, R in eq. (42)) one immediately finds out the charges of the fields: $\Psi_a, \bar{\Psi}_a, \rho_a, \gamma_a, \lambda_a$ and η_a have R -charges equal to $\frac{1}{2}, -\frac{1}{2}, -1, 1, -\frac{1}{2}$ and $\frac{1}{2}$, respectively, whereas the bosonic currents J_a and \bar{J}_a are neutral with respect to R . This means that the superconformal currents (Φ_a, \mathcal{J}_a) and their BRST ancestors (λ_a, ρ_a) have well-defined R -charge $((\frac{1}{2}, 0)$ for the (Φ_a, \mathcal{J}_a) currents and $(-\frac{1}{2}, -1)$ for their topological partners (λ_a, ρ_a)). Moreover the OPE's of G with (Φ_a, \mathcal{J}_a) and (λ_a, ρ_a) are

$$\begin{aligned} G(z_1)\Phi_a(z_2) &= \frac{1}{2} \frac{\lambda_a(z_2)}{(z_1 - z_2)^2} + \frac{\partial \lambda_a(z_2)}{z_1 - z_2} \\ G(z_1)\mathcal{J}_a(z_2) &= \frac{\rho_a(z_2)}{(z_1 - z_2)^2} + \frac{\partial \rho_a(z_2)}{z_1 - z_2} \\ G(z_1)\lambda_a(z_2) &= G(z_1)\rho_a(z_2) = 0 \ , \end{aligned} \tag{49}$$

which confirms our previous conclusion that the currents have the right transformation properties under the topological symmetry: the singular expansions of eq. (49) are the ones expected for the products of the topological partner of the energy-momentum tensor and the superconformal currents. On the other hand, after some calculation one can conclude that no singularity appears when the dimension-3 operators W and V are multiplied by the superconformal currents. Altogether this implies that the currents are primary with respect to the whole set of generators of

the topological algebra and therefore the topological and supercurrent symmetries are indeed compatible.

We finally turn our attention to the relationship between the topological and supersymmetry structures of our model. First of all we notice that the supersymmetry generators T_F^+ , T_F^- , R_F , V_F and W_F all possess a well-defined topological $U(1)$ quantum number: they transform under R as fields with charges $-\frac{1}{2}$, $\frac{1}{2}$, 0 , $\frac{3}{2}$ and 1 respectively. However, the $N = 1$ supersymmetry generator T_F does not transform as an eigenstate of the R -current. Indeed, a glance at eq. (32) reveals that T_F splits into three contributions, each of them with a well-defined R -charge and which are precisely the generators of the extended supersymmetry algebra. This fact means that the topological symmetry can be considered as responsible for the enlargement of the supersymmetry that is produced at the topological point of the matter + ghost system. Moreover, all the generators of the extended supersymmetry are BRST trivial, *i.e.* there exist new fields \mathcal{T}_F^\pm , \mathcal{R}_F , \mathcal{W}_F and \mathcal{V}_F such that

$$\begin{aligned} T_F^\pm &= \delta \mathcal{T}_F^\pm & R_F &= \delta \mathcal{R}_F \\ W_F &= \delta \mathcal{W}_F & V_F &= \delta \mathcal{V}_F . \end{aligned} \tag{50}$$

The explicit expressions for the new fields appearing in eq. (50) are

$$\begin{aligned} \mathcal{T}_F^+ &= - \frac{i}{2\sqrt{2(\kappa+h)}} \bar{\Psi}_a (\rho_a + \Lambda_a) \\ \mathcal{T}_F^- &= \frac{1}{2(\kappa+h)} \lambda_a \bar{J}_a \\ \mathcal{R}_F &= - \frac{i}{\sqrt{2(\kappa+h)}} \bar{\Psi}_a \lambda_a \\ \mathcal{W}_F &= - \frac{1}{[2(\kappa+h)]^{\frac{3}{2}}} \Psi_a (f^{abc} J_b \lambda_c + 2ih\partial\lambda_a) \\ \mathcal{V}_F &= - \frac{i}{6(\kappa+h)} f^{abc} \Psi_a \Psi_b \lambda_c . \end{aligned} \tag{51}$$

The BRST-exactness of the supersymmetry generators implies that all states created by acting on the vacuum with a finite number of them can be gauged away.

In other words, the extended supersymmetry of our model is topological, and the cohomology of Q encodes the global information about the supersymmetric Hilbert space.

In order to achieve a better understanding of the relationship between the supersymmetric and topological symmetries of our model, let us study how the BRST current Q acts on T_F^\pm and R_F . A straightforward calculation gives the result

$$\begin{aligned} Q(z_1)T_F^+(z_2) &= 0 \\ Q(z_1)T_F^-(z_2) &= -\frac{I(z_2)}{(z_1 - z_2)^2} \\ Q(z_1)R_F(z_2) &= 0, \end{aligned} \tag{52}$$

where I is a new dimension- $\frac{1}{2}$ bosonic current whose explicit expression is

$$I = i\sqrt{2(\kappa + h)}\gamma_a\Psi_a + \frac{i}{2}f^{abc}\gamma_a\gamma_b\lambda_c. \tag{53}$$

Moreover, when Q acts on \mathcal{T}_F^\pm and \mathcal{R}_F , their topological partners are obtained as residue of the simple pole singularity, while some extra contributions appear in the double pole. One has:

$$\begin{aligned} Q(z_1)\mathcal{T}_F^+(z_2) &= \frac{T_F^+(z_2)}{z_1 - z_2} \\ Q(z_1)\mathcal{T}_F^-(z_2) &= \frac{\mathcal{I}(z_2)}{(z_1 - z_2)^2} + \frac{T_F^+(z_2)}{z_1 - z_2} \\ Q(z_1)\mathcal{R}_F(z_2) &= -\frac{d}{(z_1 - z_2)^2} + \frac{R_F(z_2)}{z_1 - z_2}, \end{aligned} \tag{54}$$

where d is the dimension of the topological algebra (*i.e.*, $d = \dim g$ as in eqs. (41)-(43)), and \mathcal{I} is a dimension- $\frac{1}{2}$ fermionic field given by

$$\mathcal{I} = -\gamma_a\lambda_a. \tag{55}$$

It can be readily checked that \mathcal{I} is the BRST ancestor of I , *i.e.*,

$$Q(z_1)\mathcal{I}(z_2) = \frac{I(z_2)}{z_1 - z_2}. \tag{56}$$

We thus see that, in trying to relate T_F^\pm and R_F to their BRST partners, we are forced to introduce a new BRST doublet of fields (\mathcal{I}, I) . Actually, a whole plethora of new operators appear when the generators of the topological algebra, on the one hand, and those of the supersymmetric one, on the other, are multiplied together. The reason for this proliferation of operators can be traced back to the non-nilpotency of one of the generators of the extended supersymmetric algebra (*i.e.* of T_F^-). Indeed one would expect that all operators appearing in the symmetry algebra could be arranged into supersymmetry multiplets whose components are generated by acting with T_F^+ and T_F^- . As $(T_F^-)^2 \neq 0$, many new components of the supersymmetry multiplet are generated in this process. However when g is taken to be an abelian algebra (*i.e.*, when $f^{abc} = h = 0$), this problem disappears since our system possesses an $N = 2$ supersymmetry. One can then easily check in this case that the full algebra closes with the sole addition of I and its BRST partner \mathcal{I} to the generators of the topological and supersymmetry algebras.

Let us consider now the question of the uniqueness of our construction. We could try to modify the matter sector by considering more matter species (*i.e.*, by taking $M > 2$ in eqs. (17)-(19)). It is immediate to see that, in this case, the vanishing of the central charge c does not follow from the zero level condition $x = 0$. Of course we could adjust the levels k_i in such a way that $c = 0$, but we still must demand the BRST exactness of \mathcal{J}_a and Φ_a (see eq. (34)) and of T . This latter condition means that we ought to be able to find an operator G such that $T = \delta G$. It can be easily concluded that, when $M > 2$, it is not possible to find an expression for G local in the currents. The proof of this statement is the same as in the bosonic case (see ref. [3]) and will not be reproduced here. This result implies that our construction only works for $M = 2$. At this point it is interesting to recall [3] that the bosonic topological current algebras can be defined for $M = 1, 2$, contrary to what happens in the present supersymmetric case in which $M = 1$ is not allowed.

The realization of the extended topological symmetry we have found admits deformations, *i.e.*, redefinitions of its generators such that the transformed oper-

ators satisfy the same algebra. Suppose that α_a are a set of c-number constants, and let us redefine T , G and R as follows:

$$\begin{aligned} T &\rightarrow T + \sum_a \alpha_a \partial \mathcal{J}_a \\ G &\rightarrow G + \sum_a \alpha_a \partial \rho_a \\ R &\rightarrow R + \sum_a \alpha_a \mathcal{J}_a , \end{aligned} \tag{57}$$

while Q , V and W are not transformed. It's not difficult to see that the redefined fields still close the extended topological algebra for the same value of the parameter d (*i.e.* for $d = \dim g$). Of course, this transformation does not preserve the supercurrent symmetry, but it does preserve the supersymmetric Kazama algebra if we redefine T_F^\pm and R_F as

$$\begin{aligned} T_F^+ &\rightarrow T_F^+ \\ T_F^- &\rightarrow T_F^- + 2 \sum_a \alpha_a \partial \Phi_a \\ R_F &\rightarrow R_F + 2 \sum_a \alpha_a \mathcal{J}_a . \end{aligned} \tag{58}$$

Let us now recapitulate our main results. We have been able to find a topological conformal model possessing a superconformal current algebra compatible with the topological symmetry, in which the generators of the current algebra are exact in the BRST cohomology defined by the topological symmetry. The topological algebra closes only after the introduction of two dimension-3 operators (W and V in eqs. (45) and (46)). The compatibility of the supersymmetry with the topological symmetry of the model requires the extension of the former. The generators of the extended supersymmetric algebra include three dimension-3 fermionic operators (T_F^\pm , V_F), a dimension-1 $U(1)$ current (R_F) and a dimension-2 bosonic field (W_F). All these generators are BRST exact, a fact which exhibits the topological nature of the supersymmetric algebra.

Our results raise several questions that deserve further study. First of all, we would like to point out that the model we have constructed is the supersymmetric analogue of the G/G coset model. The states of the G/G topological field theory are intimately connected with the conformal blocks of the Wess-Zumino-Witten theory for the group G [12,13], so it's natural to conjecture that the states of our model are in correspondence with the conformal blocks of the supersymmetric WZW model. Furthermore it has been shown [12,13,14] that, after a deformation such as the one in eq. (57), the physical states of the theory are equivalent to those of the minimal models coupled to gravity (*i.e.*, to those of the non-critical string theory). Therefore, in our case, one would expect to get a relationship of the deformed theory with the non-critical superstrings. On the other hand, the $SL(N)/SL(N)$ models have been obtained in ref. [4] from the WZW model based on the $GL(N,N)$ supergroup. From this fact one is led to suspect that the model constructed above could be described as a suitable supersymmetrization of the $GL(N,N)$ WZW theory. We expect to analyse these topics in the near future.

Acknowledgements: The authors would like to thank H.L. Hu, J.M.F. Labastida, P.M. Llatas, J. Mas, and J. Sánchez de Santos for discussions. One of us (J.M.I.) is grateful to the Department of Physics of Queen Mary and Westfield College, where the last part of this work was carried out, for hospitality, and to Conselleria de Educacion da Xunta de Galicia for financial support. This work was supported in part by DGICYT under grant PB90-0772, and by CICYT under grants AEN90-0035 and AEN93-0729.

REFERENCES

1. E. Witten, *Comm. Math. Phys.* **117**(1988), 353.
2. For a review see D. Birmingham, M. Blau, M. Rakowski and G. Thompson, *Phys. Rep.* **209**(1991), 129.
3. J. M. Isidro and A. V. Ramallo, *Phys. Lett.* **B316**(1993), 488.
4. J. M. Isidro and A. V. Ramallo, *Nucl. Phys.* **B414**(1994), 715.
5. T. Eguchi and S.-K. Yang, *Mod. Phys. Lett.* **A4**(1990), 1653; T. Eguchi, S. Hosono and S.-K. Yang, *Comm. Math. Phys.* **140**(1991), 159.
6. W. Lerche, C. Vafa and N.P. Warner, *Nucl. Phys.* **B324**(1989), 427.
7. P. Di Vecchia, V.G. Knizhnik, J.L. Petersen and P. Rossi, *Nucl. Phys.* **B253**(1985), 701.
8. V.G. Kac and I.T. Todorov, *Comm. Math. Phys.* **102**(1985), 337.
9. Y. Kazama, *Mod. Phys. Lett.* **A6**(1991), 1321.
10. E. Getzler, “Manin pairs and topological field theory”, MIT preprint(1993) (hep-th/9309057).
11. E. Witten, *Comm. Math. Phys.* **144**(1992), 189.
12. M. Spiegelglas and S. Yankielowicz, *Nucl. Phys.* **393**(1993), 301.
13. O. Aharony et al., *Nucl. Phys.* **B399**(1993), 527, *Phys. Lett.* **B289**(1992), 309, *Phys. Lett.* **B305**(1993), 35.
14. H.L. Hu and M. Yu, *Phys. Lett.* **B289**(1992), 302, *Nucl. Phys.* **B391**(1993), 389.